Vibrations and Waves

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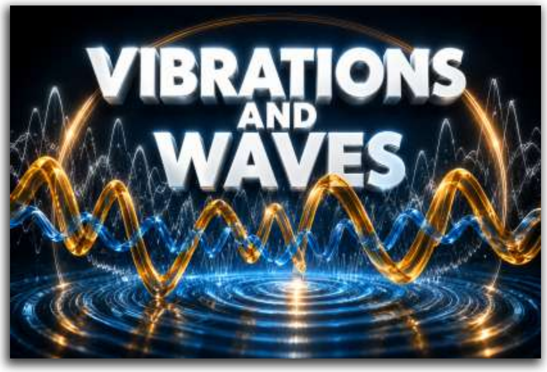

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Objectives

At the end of this course the student should:

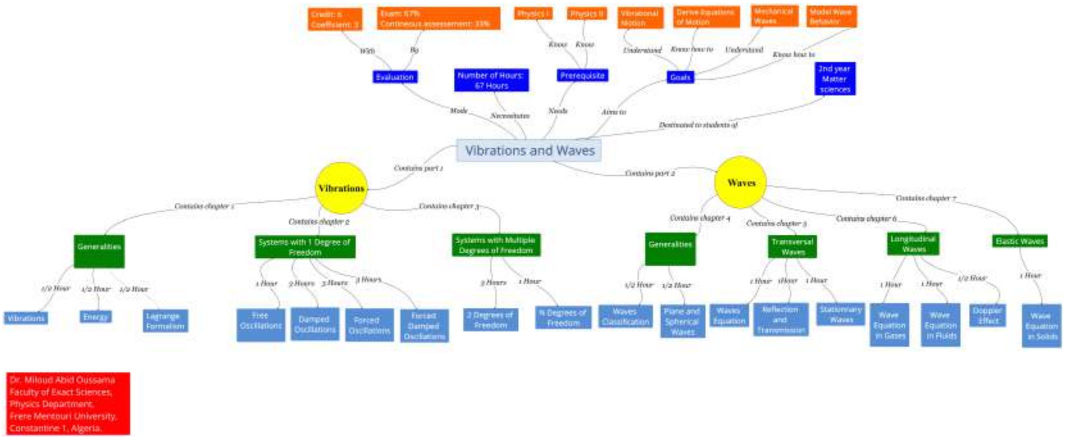
- **Understand** the fundamental concepts of oscillations (definition, periodicity, parameters).
- **Apply** the equations of motion for a harmonic oscillator.
- **Analyze** the effects of damping and external forces.
- **Evaluate** resonance conditions and their consequences.
- **Apply** wave equations in different media.
- **Analyze** reflection, transmission, and interference phenomena.
- **Create** simplified models of vibrational or wave systems.

Introduction

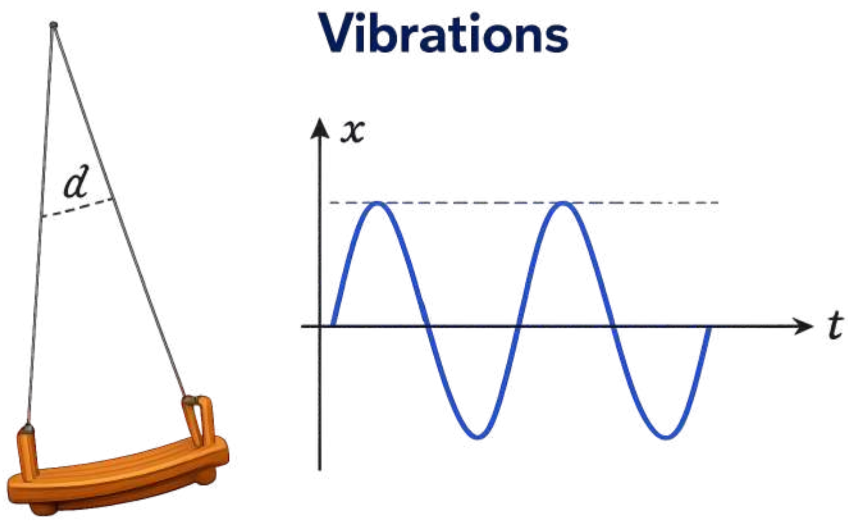
The Course "**Vibrations and Waves**" is destinated to 2nd Year Matter Sciences students.

Dr. Miloud Abid Oussama, PhD in Computational Material Sciences at FMU Constantine 1 is the instructor of this course.

The course is divided into two parts as described in the chart below.



1. Vibrations



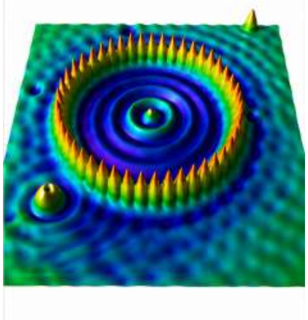
Generalities

1. Introduction

In the physical world there are many examples of things that vibrate or oscillate, i.e. perform periodic motion. Everyday examples are a swinging pendulum, a vibrating Phoneand a car bouncing up and down on its springs. The most basic form of periodic motion is called simple harmonic motion (SHM p.18).[1] p.19

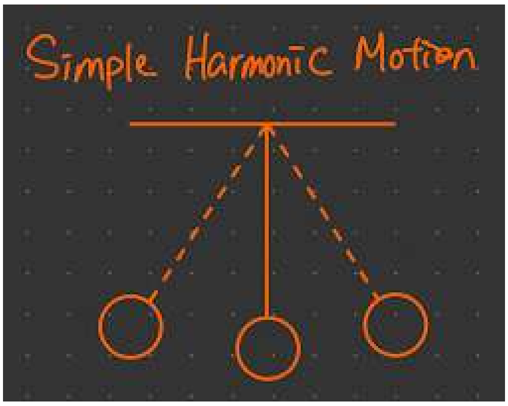
(see watch)





2. Vibrations

Definition



An ***oscillation*** *is a time-dependent motion in which a system undergoes repeated displacement about an equilibrium position due to the presence of a restoring force.*[2] p.19

More formally, let $x(t)$ denote the generalized coordinate describing the system. An oscillatory motion satisfies:

- The existence of an equilibrium position x_0 such that:

  $$F(x_0) = 0$$

- A restoring force acting toward equilibrium when the system is displaced:

  $$F(x) \approx -k(x - x_0), \quad k > 0$$

Under these conditions, the system exhibits periodic or quasi-periodic motion.

Oscillations arise from the competition between:

- **Inertia**, which tends to maintain motion
- **Restoring forces**, which drive the system back toward equilibrium

This interplay leads to continuous energy exchange between:

- **Kinetic energy**
- **Potential energy**

Example

1. **Simple Pendulum**

   A mass suspended from a pivot undergoes angular oscillations under gravity.

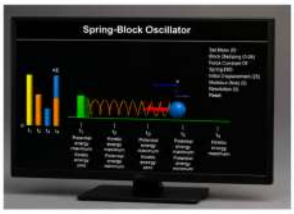
2. **Mass–Spring System**

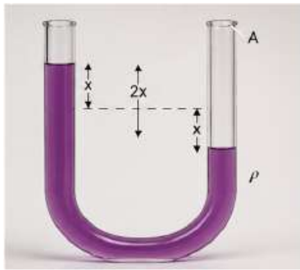
   A mass attached to a spring obeys Hooke's law:

   $F = -kx$

3. **Floating Cylinder in a Fluid**

   Vertical oscillations arise due to buoyant restoring forces.





2.1. Linear Approximation and Harmonic Motion

Near equilibrium, most physical systems can be approximated by a linear restoring force:

$F(x) \approx -k(x - x_0)$

Applying Newton's second law:

$m\,\dfrac{d^2x}{dt^2} = -k(x - x_0)$

which leads to the equation of **simple harmonic motion (SHM)**:

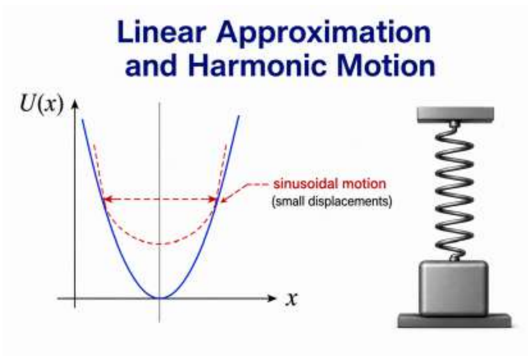
$m\,\dfrac{d^2x}{dt^2} + kx = 0$

The general solution is:

$x(t) = A\cos(\omega t + \phi)$

where:

- A is the amplitude
- ϕ is the phase constant
- $\omega = \sqrt{\dfrac{k}{m}}$ is the angular frequency



2.2. Key Characteristics of Oscillatory Motion

- **Period**:

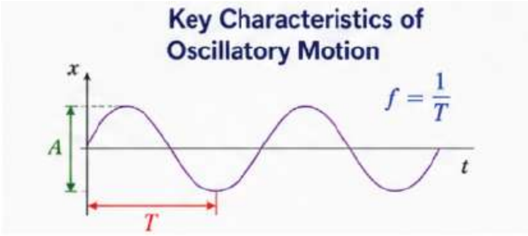
  $T = \frac{2\pi}{\omega}$

- **Frequency**:

  $f = \frac{1}{T}$

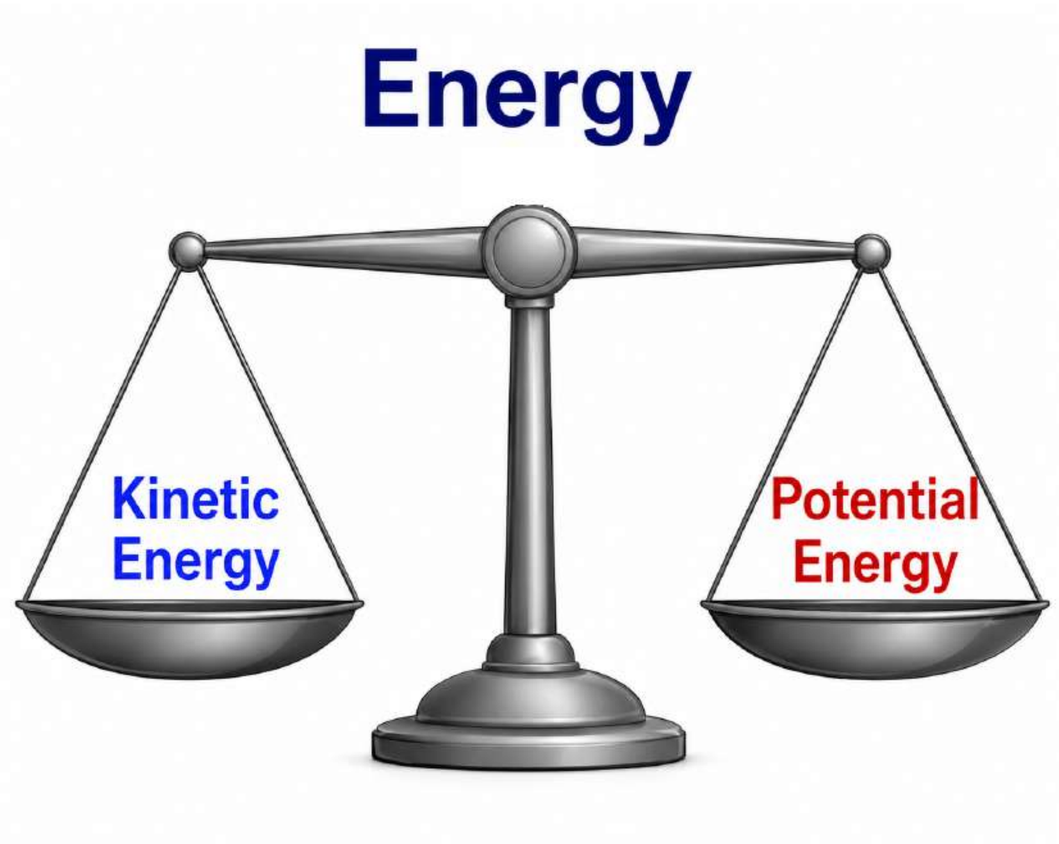
- **Energy Conservation (undamped case)**:

  $E = \frac{1}{2}kA^2 = \text{constant}$



3. Energy

Fundamental



Energy provides a powerful and unified framework for analyzing mechanical systems, particularly oscillatory motion. In classical mechanics, the total mechanical energy is expressed as the sum of **kinetic** and **potential** energies.

3.1. Kinetic Energy

Kinetic energy (T [p.18])is associated with motion and depends on the type of movement.

a) *Translational Motion:*

For a particle of mass m moving with velocity $\mathbf{v}$:

$T = \frac{1}{2} m v^2$

b) *Rotational Motion:*

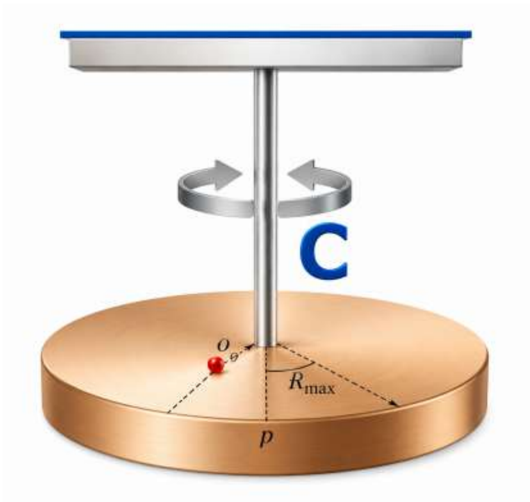
For a rigid body rotating about a fixed axis with angular velocity (ω):

$T = \frac{1}{2} J \omega^2$

where:

- J is the moment of inertia with respect to the rotation axis:

  $J = \sum_i m_i r_i^2$



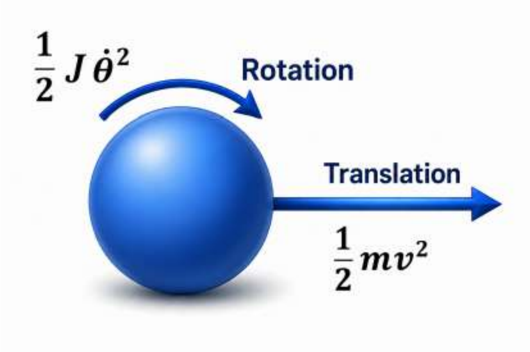
c) *Combined Translation and Rotation:*

For a rigid body undergoing general motion:

$$T = \frac{1}{2}mv_{\text{cm}}^2 + \frac{1}{2}J_{\text{cm}}\omega^2$$

where:

- v_{cm} : velocity of the center of mass
- J_{cm} : moment of inertia about the center of mass

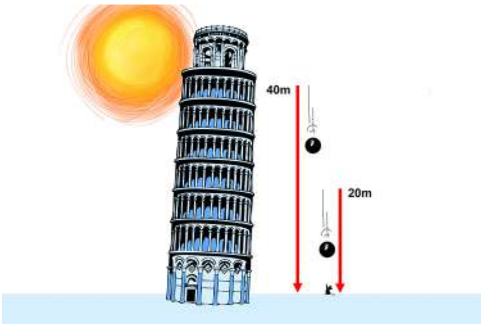


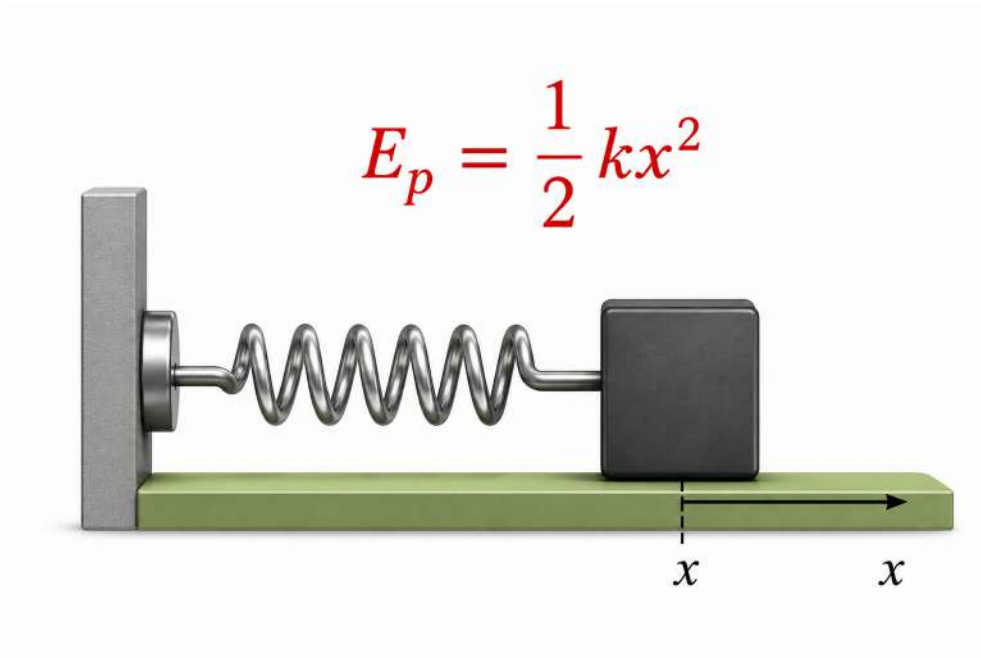
3.2. Potential Energy

Potential energy (V p.18) is associated with the **configuration** of a system and arises from conservative forces.

A force $\mathbf{F}$ is conservative if it can be derived from a scalar potential V :

$$\mathbf{F} = -\nabla V$$



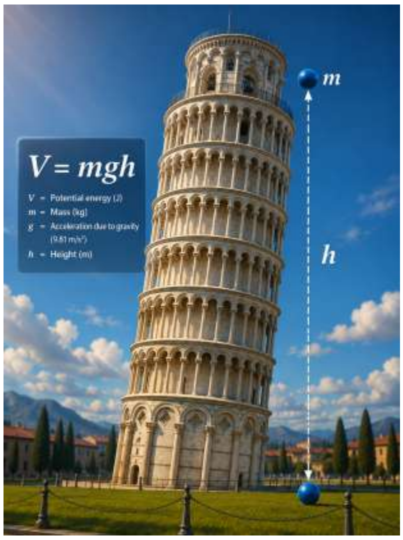


a) Gravitational Potential Energy (Near Earth's Surface)

$$V = mgh$$

where:

- h is the vertical displacement
- g is the gravitational acceleration



b) Elastic (Spring) Potential Energy

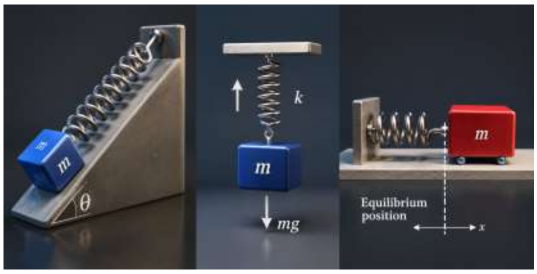
For a linear restoring force obeying Hooke's law:

$$F = -kx$$

the associated V p.18 is:

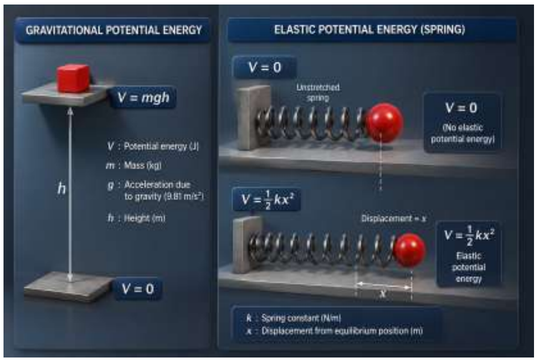
$$V(x) = \frac{1}{2} kx^2$$

This quadratic dependence is fundamental in oscillatory systems.

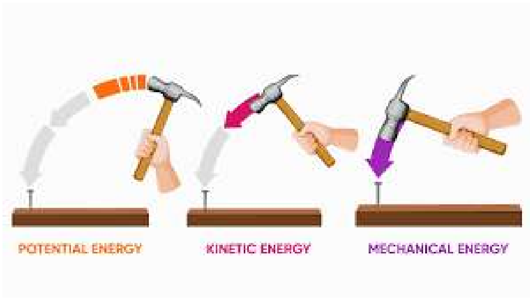


Extra

- Potential energy is defined up to an additive constant
- Only differences in potential energy have physical meaning
- Stable equilibrium corresponds to a **minimum of (V(x))**



3.3. Total Mechanical Energy



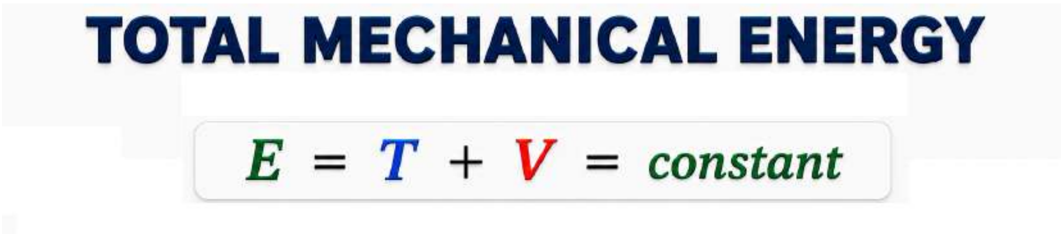
The total energy of a system (E [p.18]) is:

$E = T + V$

For conservative systems:

$E = \text{constant}$

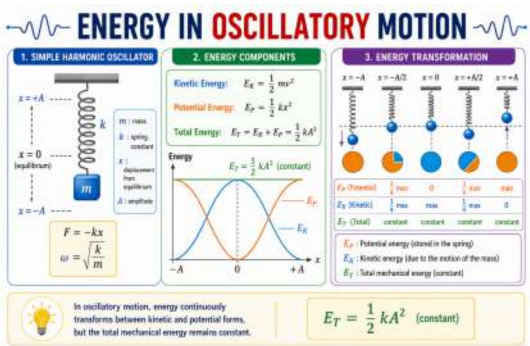
This principle provides an alternative to Newton's laws and is especially powerful for solving vibration problems.



3.4. Energy in Oscillatory Motion

In oscillatory systems (e.g., mass–spring):

- At maximum displacement:
-
  - $T = 0$
  - $V = \frac{1}{2}kA^2$
- At equilibrium:
-
  - $V = 0$
  - $T = \frac{1}{2}kA^2$



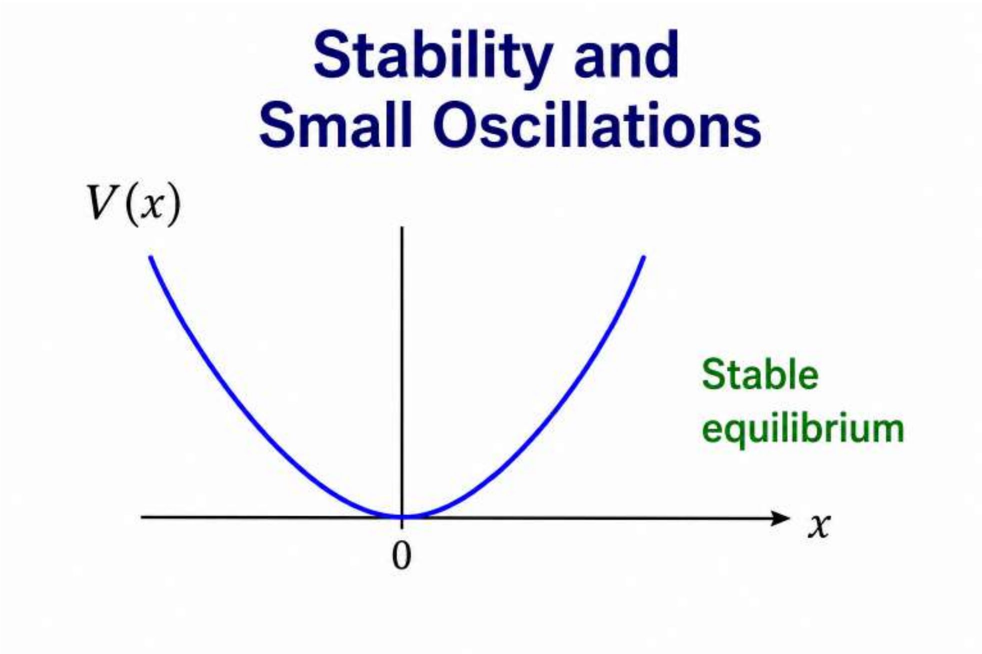
Energy continuously oscillates between kinetic and potential forms while the total remains constant.[3] [p.19]

3.5. Stability and Small Oscillations

Near equilibrium x_0, the potential can be expanded:

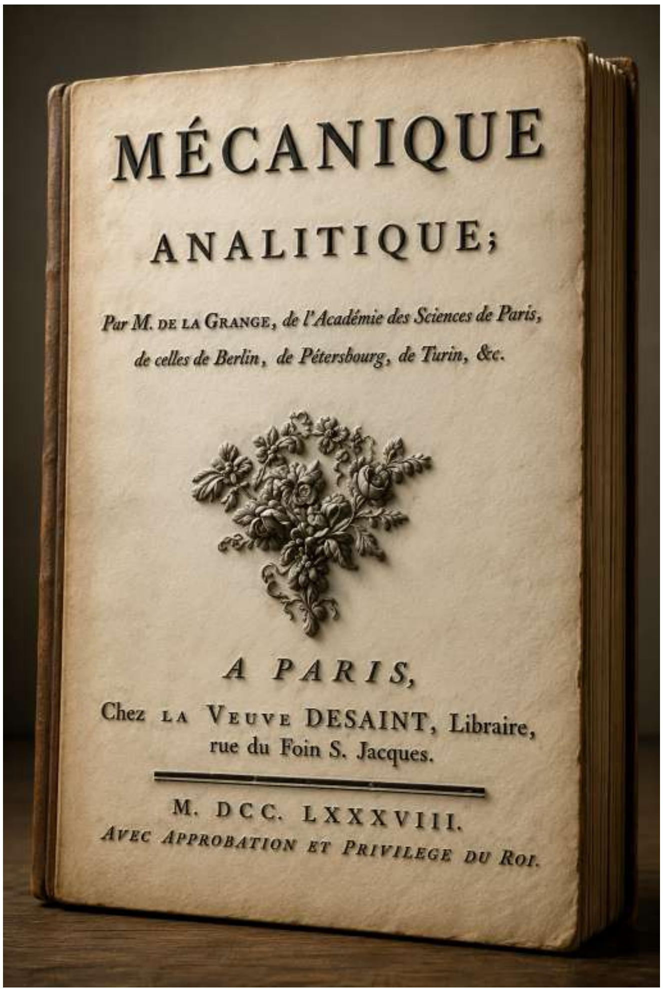
$V(x) \approx V(x_0) + \frac{1}{2}k(x - x_0)^2$

This leads naturally to **simple harmonic motion**, showing that many physical systems behave as oscillators near stable equilibrium.



4. Lagrange Formalism

The Lagrangian formalism provides a powerful and general framework for analyzing the dynamics of mechanical systems. Unlike Newtonian mechanics, it is based on **energy principles** and is particularly well-suited for systems with constraints and multiple degrees of freedom (*DoF* p.18).



4.1. Generalized Coordinates

A mechanical system can be described using a set of **generalized coordinates**:

$q_1, q_2, \ldots, q_N$

These are independent variables that uniquely define the configuration of the system at any time t .

- They may be **Cartesian coordinates**, angles, or any convenient parameters
- They automatically incorporate constraints of the system

The corresponding **generalized velocities** are:

$\dot{q}_i = \frac{dq_i}{dt}$

Example

1. **Particle in 3D space**

   The position of a point (M) is described by:

   (x, y, z)

   → 3 generalized coordinates
2. **Rigid body in space**

   A rigid body requires **6 coordinates**:
   - 3 for the center of mass position
   - 3 for orientation (e.g., Euler angles)

4.2. Degrees of Freedom

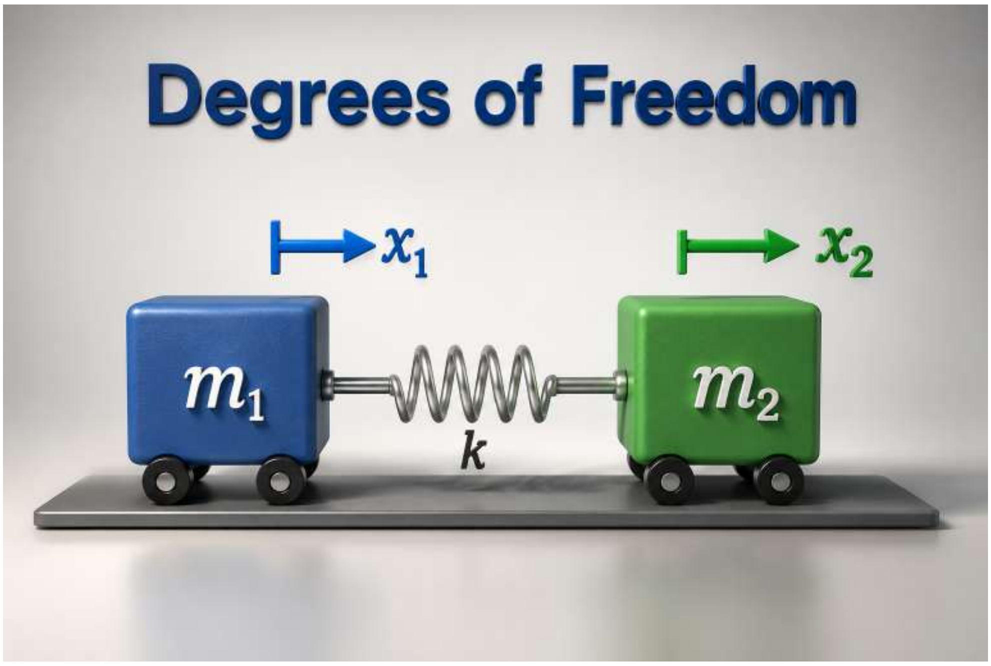
The **number of degrees of freedom** (***DoF*** p.18) is the number of independent generalized coordinates required to describe the system:

$N = \text{number of independent } q_i$

With constraints

If constraints exist:

$N = N_{\text{total}} - N_{\text{constraints}}$



Example

Consider two particles connected by a rigid rod of fixed length (L):

- Total coordinates: $(2 \times 3 = 6)$
- Constraint: distance fixed

  $|\mathbf{r}_1 - \mathbf{r}_2| = L$

→ Number of degrees of freedom:

$N = 6 - 1 = 5$

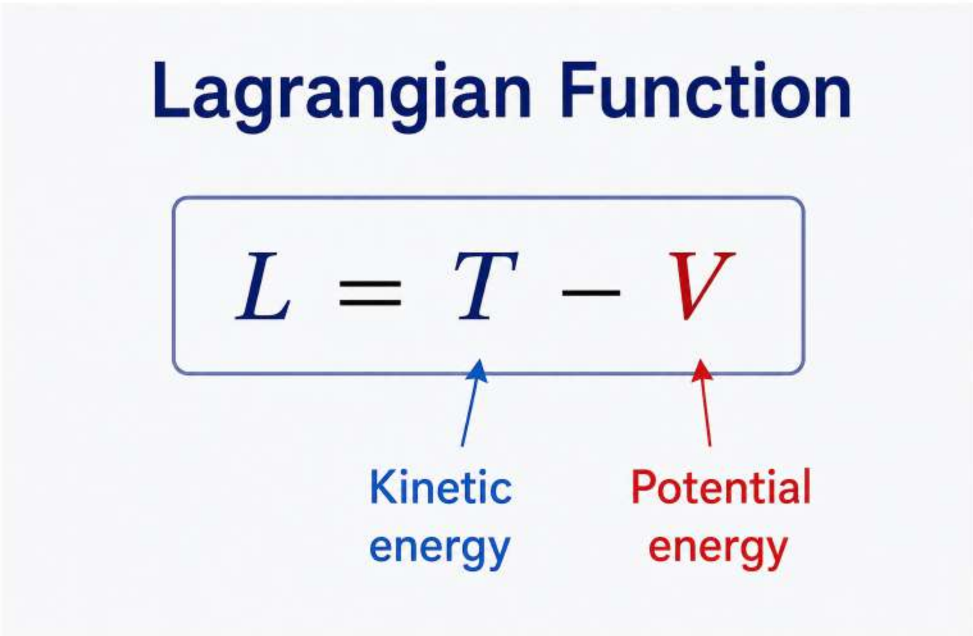
4.3. Lagrangian Function

*The dynamics of a system are determined by the **Lagrangian**:*[4] p.19

$L(q_i, \dot{q}_i, t) = T - V$

where:

- T : kinetic energy
- V : potential energy



4.4. Euler–Lagrange Equations

The equations of motion are obtained from the **principle of least action**:

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$

These are the **Euler–Lagrange equations**, one for each generalized coordinate q_i .

4.5. Special Cases

a) One Degree of Freedom (N = 1)

For a single coordinate q :

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0$

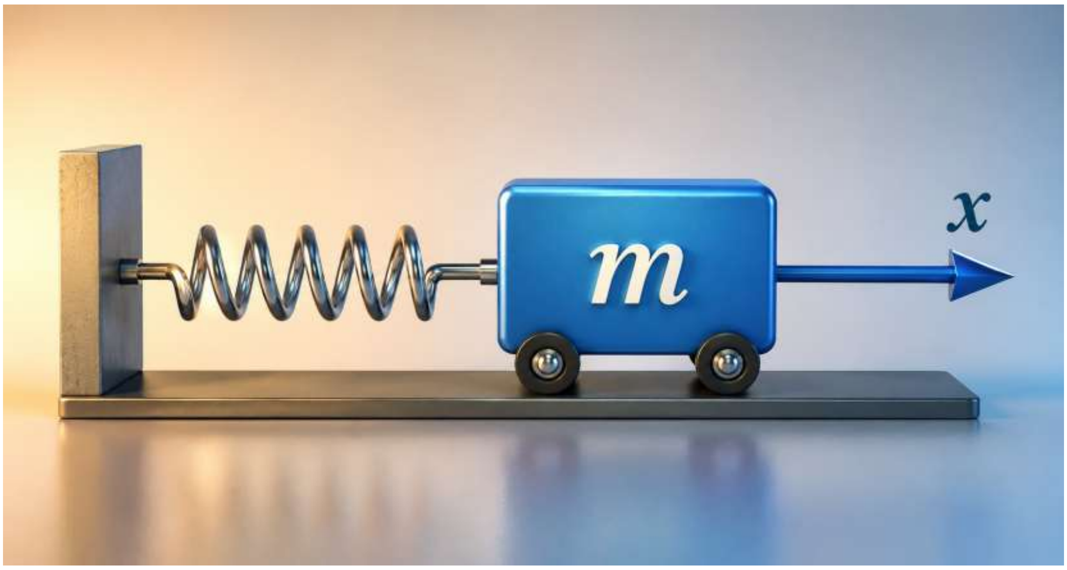
b) One-Dimensional Motion

If $q = x$:

$\frac{d}{dt}(m\dot{x}) + \frac{\partial V}{\partial x} = 0$

which reduces to Newton's second law:

$m\ddot{x} = -\frac{dV}{dx}$



c) Rotational Motion

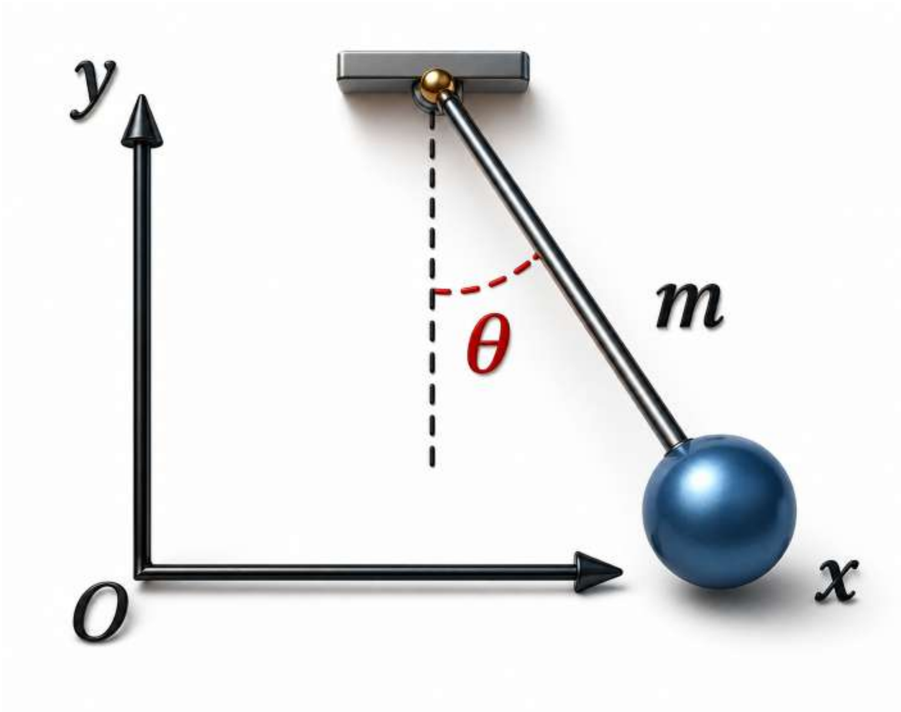
For a rotational coordinate θ :

- Kinetic energy:

  $T = \frac{1}{2}J\dot{\theta}^2$

- Lagrange equation:

  $J\ddot{\theta} = -\frac{dV}{d\theta}$



4.6. Advantages of the Lagrangian Formalism

- Handles **constraints naturally**
- Works in **any coordinate system**
- Easily generalized to:
-
    - multi-particle systems
    - continuous systems (fields)
- Forms the basis of:
-
    - quantum mechanics
    - field theory
    - advanced solid-state physics

Abbreviation

DoF : number of degrees of freedom
E : Total Energy
SHM : Simple Harmonic Motion.
T : Kinetic Energy
V : Potential Energy

Bibliography

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