Vibrations and Waves

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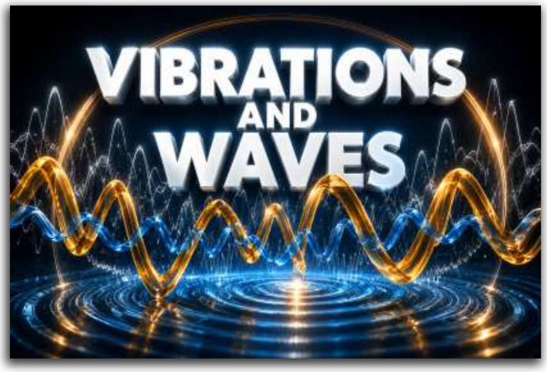

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Objectives

At the end of this course the student should:

- **Understand** the fundamental concepts of oscillations (definition, periodicity, parameters).
- **Apply** the equations of motion for a harmonic oscillator.
- **Analyze** the effects of damping and external forces.
- **Evaluate** resonance conditions and their consequences.
- **Apply** wave equations in different media.
- **Analyze** reflection, transmission, and interference phenomena.
- **Create** simplified models of vibrational or wave systems.

System with 1 degree of freedoom

1. Free Motion

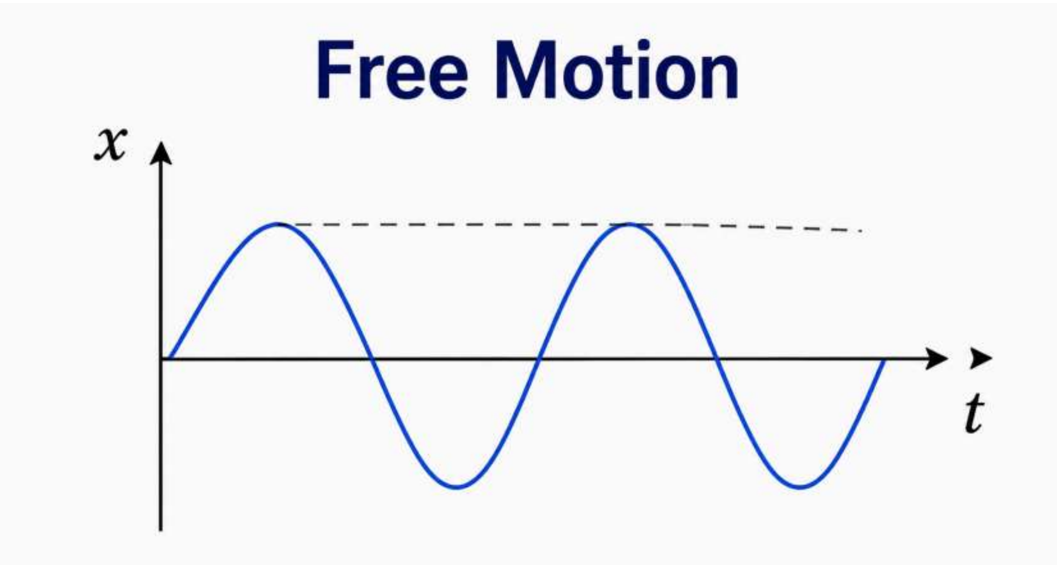
Definition

A system is said to undergo **free motion** when it evolves without external forces or dissipative effects. The motion is governed solely by its initial conditions and intrinsic properties.

Mathematically:

$$\mathbf{F}_{\text{ext}} = 0$$

1.1. Free Motion of a Particle



a) Newtonian Description

For a particle of mass m :

$$m\ddot{x} = 0$$

$$\ddot{x} = 0$$

Integrating:

$$\dot{x}(t) = v_0,$$

$$x(t) = x_0 + v_0 t$$

→ The motion is **uniform and rectilinear**

b) Energy Perspective

- Kinetic energy:
  $T = \frac{1}{2}mv^2 = \text{constant}$
- Potential energy:
  $V = \text{constant (or zero)}$

Thus:

$E = T + V = \text{constant}$

c) Free Motion in the Lagrangian Formalism

For a free particle:

$V = 0 \quad \Rightarrow \quad L = T = \frac{1}{2}m\dot{x}^2$

Applying Euler–Lagrange equation:

$\frac{d}{dt}(m\dot{x}) = 0$

→ Conservation of momentum:

$p = m\dot{x} = \text{constant}$

d) Free Motion of a System with Multiple Degrees of Freedom

For generalized coordinates q_i :

$L = \frac{1}{2}\sum_i m_i \dot{q}_i^2$

Euler–Lagrange equations give:

$\ddot{q}_i = 0$

Each coordinate evolves independently:

$q_i(t) = q_{i0} + v_{i0}t$

e) Free Motion in Three Dimensions

For a particle in space:

$\mathbf{r}(t) = \mathbf{r}_0 + \mathbf{v}_0 t$

Each component evolves independently:

$x(t), ; y(t), ; z(t)$

2. Damped Motion

Definition

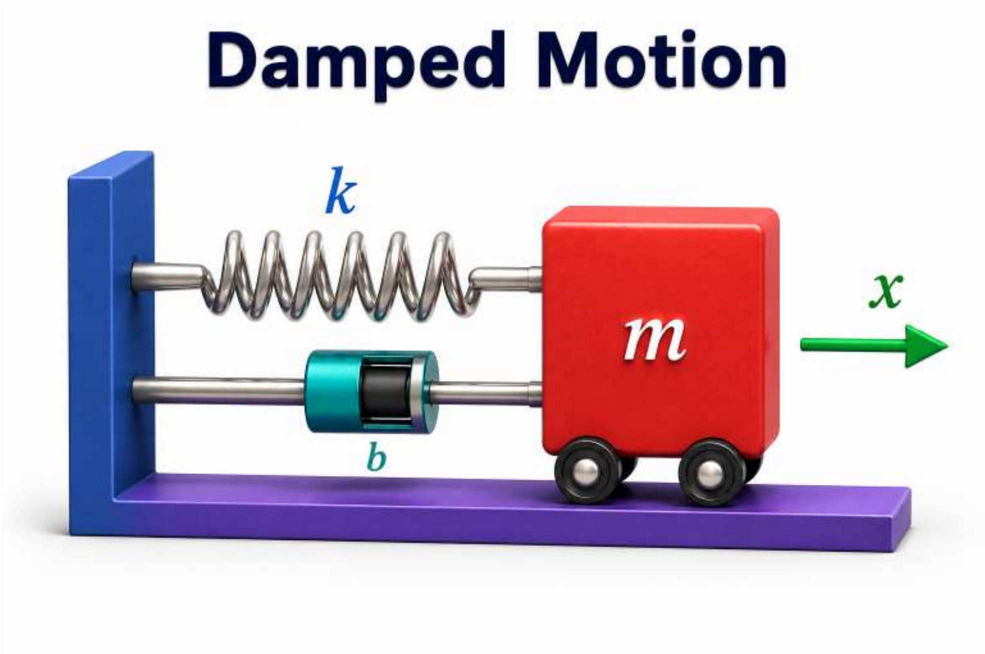
*A system undergoes **damped motion** when dissipative forces (e.g., friction or viscosity) remove energy from the system over time.*[5] p.12

The damping force is commonly modeled as proportional to velocity:

$F_d = -c\dot{x}$

where:

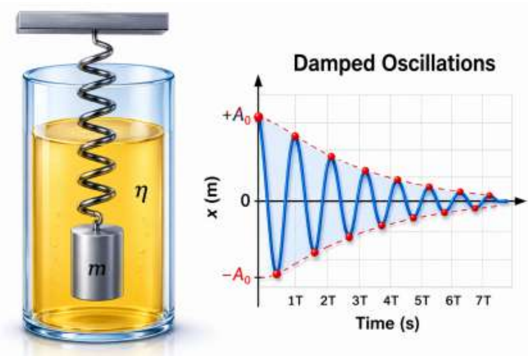
- c is the damping coefficient



2.1. Equation of Motion (EOM)

For a mass–spring–damper system *EOM* p.11:

$m\ddot{x} + c\dot{x} + kx = 0$

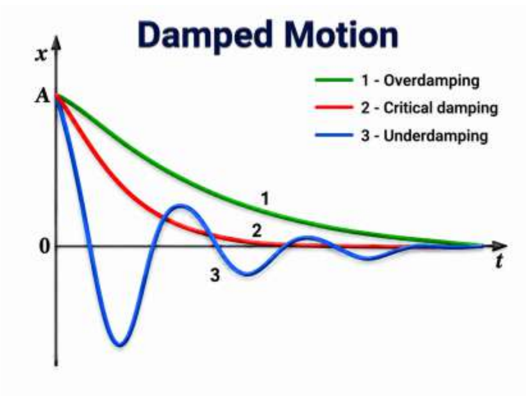


2.2. Damping Regimes

We define:

$\omega_0 = \sqrt{\frac{k}{m}}, \quad \zeta = \frac{c}{2\sqrt{mk}}$

where ζ is the **damping ratio**.



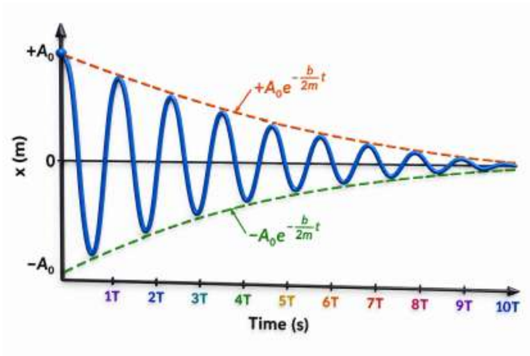
a) Underdamped ($\zeta < 1$)

- Oscillatory motion with decaying amplitude
- Solution:

  $x(t) = Ae^{-\zeta\omega_0 t}\cos(\omega_d t + \phi)$

with:

$\omega_d = \omega_0\sqrt{1-\zeta^2}$

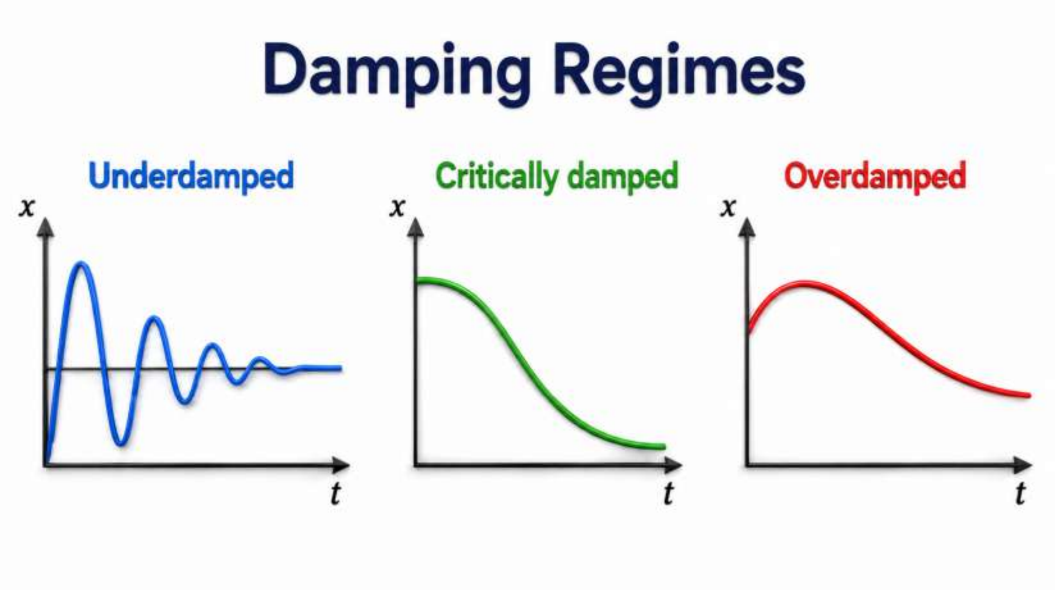


b) Critically Damped ($\zeta = 1$)

- Fastest return to equilibrium without oscillation
- No overshoot

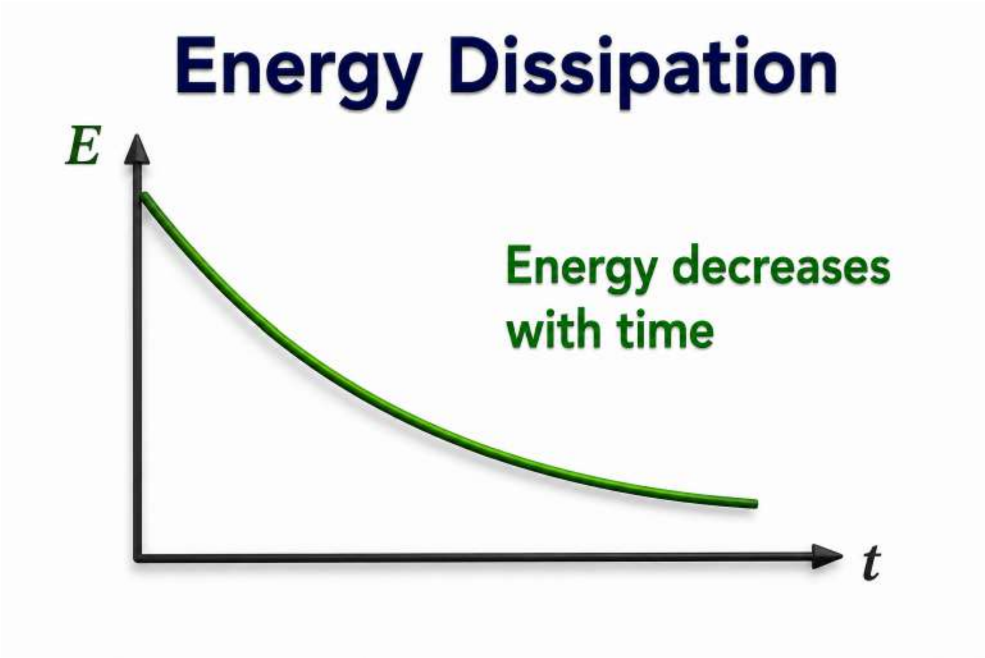
c) Overdamped ($\zeta > 1$)

- No oscillations
- Slow exponential return to equilibrium



2.3. Energy Dissipation

- Mechanical energy decreases with time:
  $E(t) \propto e^{-2\zeta\omega_0 t}$



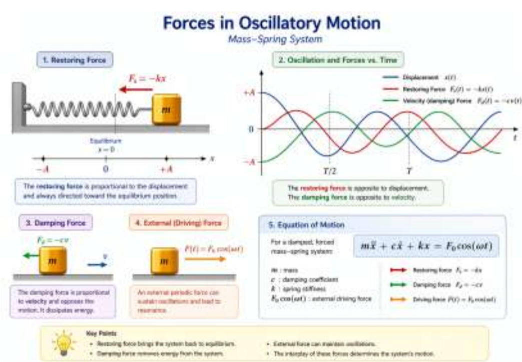
3. Forces Oscillations

Definition

A system is said to undergo **forced motion** when subjected to an external time-dependent force.

Example:

$F(t) = F_0 \cos(\omega t)$



3.1. Equation of Motion

$$m\ddot{x} + c\dot{x} + kx = F_0 \cos(\omega t)$$

3.2. General Solution

The solution is composed of two parts:

$$x(t) = x_{\text{transient}} + x_{\text{steady-state}}$$

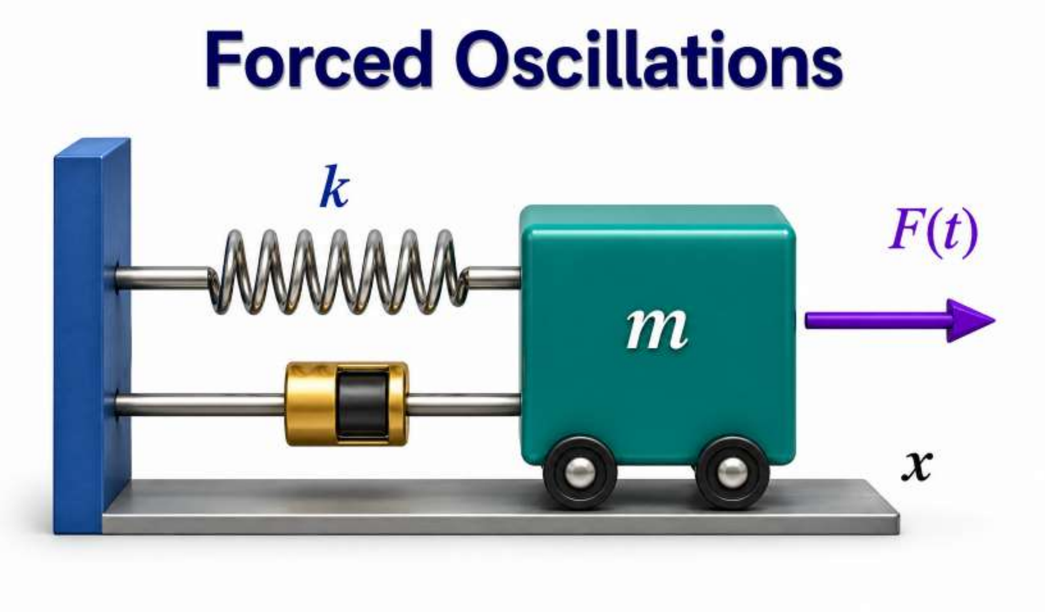
- **Transient solution**: decays over time (depends on damping)
- **Steady-state solution**: persists and oscillates at driving frequency

3.3. Steady-State Solution

$x(t) = A(\omega)\cos(\omega t - \delta)$

where:

- $A(\omega)$: amplitude
- δ : phase shift



3.4. Amplitude Response

$A(\omega) = \frac{F_0/m}{\sqrt{(\omega_0^2-\omega^2)^2+(2\zeta\omega_0\omega)^2}}$

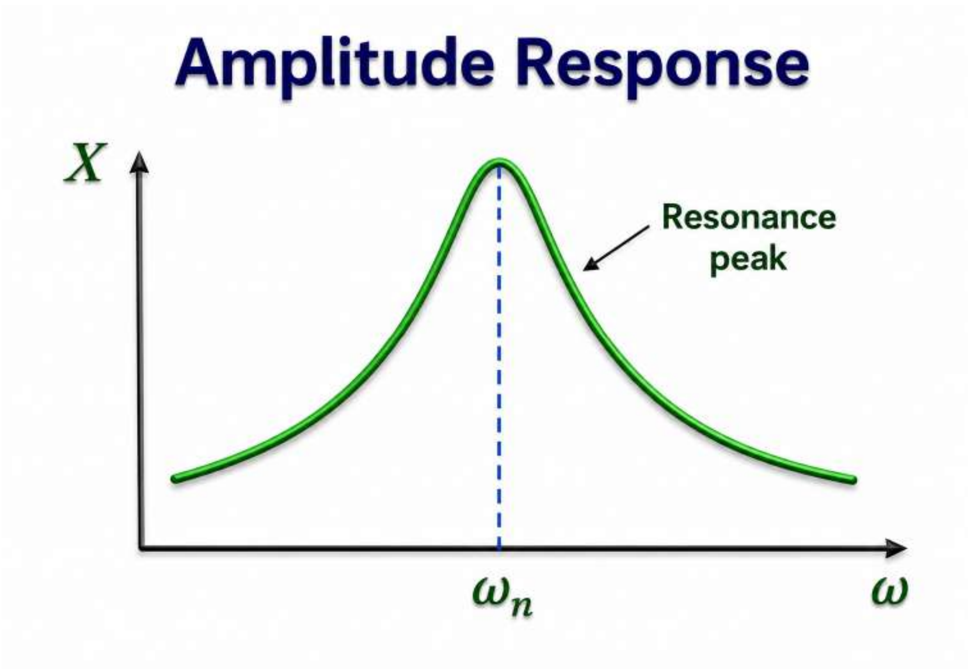
3.5. Resonance

- Occurs when amplitude is maximum
- For weak damping:

  $\omega \approx \omega_0$

At resonance:

- large amplitude oscillations
- phase shift $\delta \approx \frac{\pi}{2}$

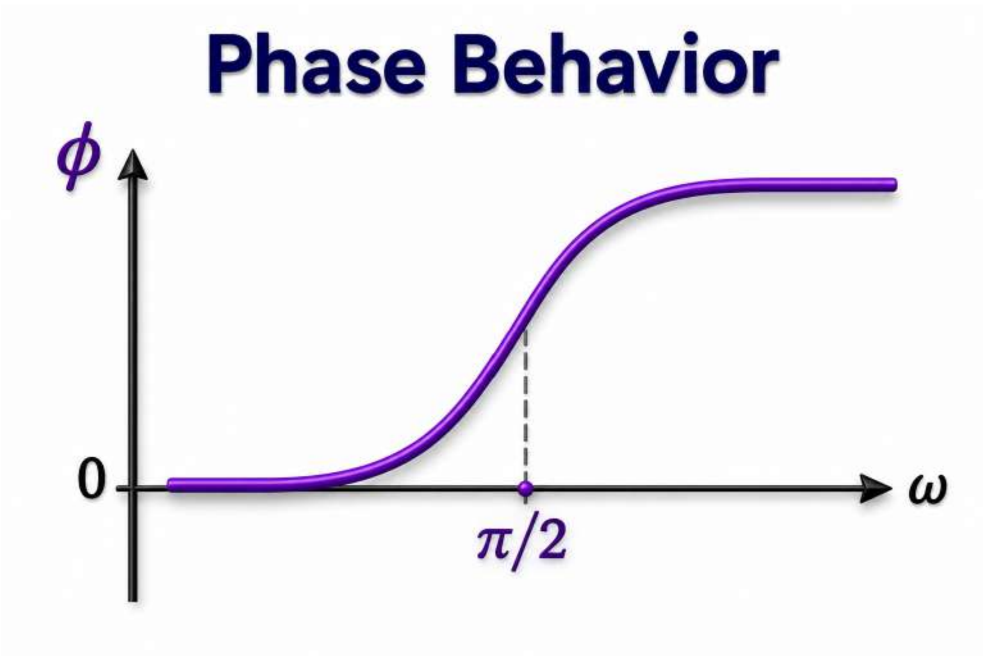


Example

(see V81nVuZ2jtQ)

3.6. Phase Behavior

- Low frequency ($\omega \ll \omega_0$):
  → motion in phase with force
- At resonance:
  → phase shift $\frac{\pi}{2}$
- High frequency ($\omega \gg \omega_0$):
  → motion out of phase



Abbreviation

EOM : Equation of Motion

Bibliography

[5] Benabadji M. K., Rappels de cours sur les Vibrations , Preparatory School, Oran, Algeria, 2016.