

Practical Work 3

Study of Friction Forces on an Inclined Plane

1. Objectives

- Understand friction forces on an inclined plane.
- Measure the static friction force (f_s) from equilibrium limits.
- Measure the dynamic friction force (f_d) using motion and travel time.
- Practice error analysis and compare results.

2. Theoretical Background

2.1 The Inclined Plane

An inclined plane forms an angle α with the horizontal. A body of mass M is subjected to:

- Weight $P = Mg$
- Normal reaction R
- Friction force f

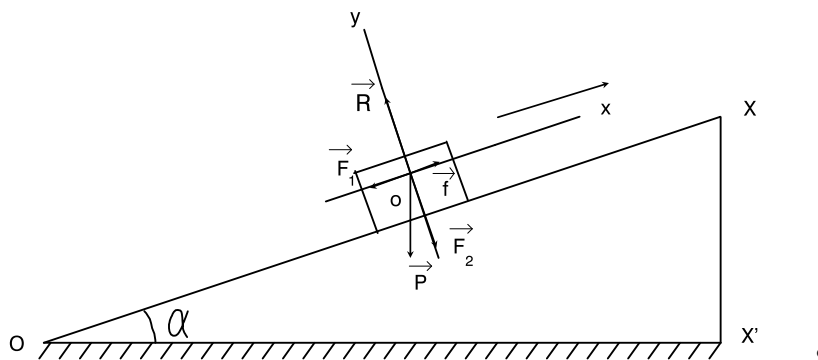


Figure 1: Inclined plane with forces

2.2 Frictional Equilibrium

- The friction force always acts in the opposite direction to the motion.
- Upwards when the body tends to move down.
- Downwards when the body tends to move up.

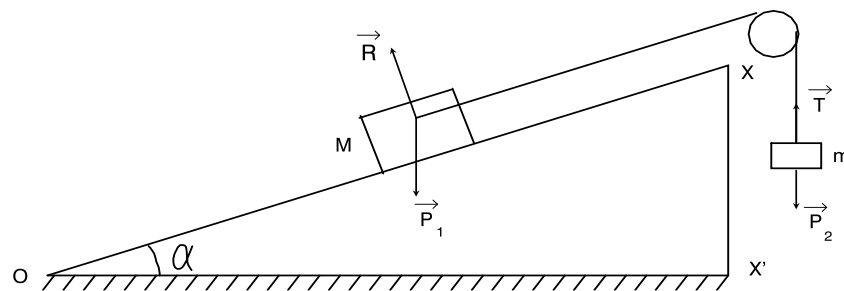


Figure 2: Equilibrium on the inclined plane

Equilibrium occurs between two limit angles:

- α_1 = lower limit (cart about to move up)
- α_2 = upper limit (cart about to move down)

At α_1 :

$$Mg \sin \alpha_1 = mg - f_s \quad \gg \quad \sin \alpha_1 = \frac{mg - f_s}{Mg} \dots\dots\dots(1)$$

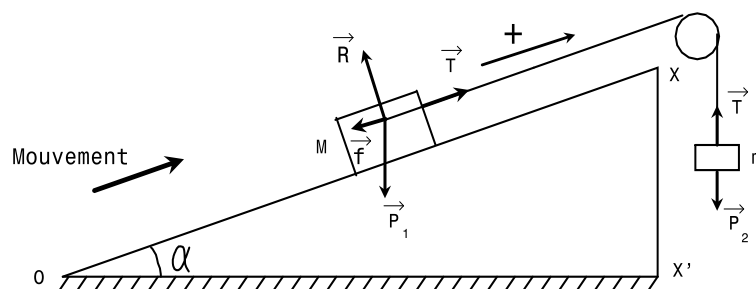


Figure 3: Lower equilibrium limit

At α_2 :

$$Mg \sin \alpha_2 = mg + f_s \quad \gg \quad \sin \alpha_2 = \frac{mg + f_s}{Mg} \dots\dots\dots(2)$$

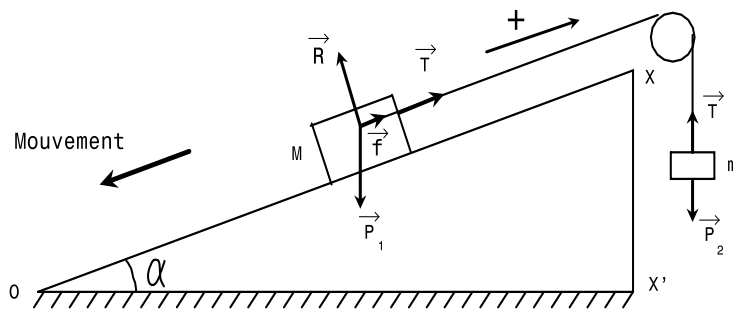


Figure 4: Upper equilibrium limit

3. Experiment 1 – Static Friction Force

Apparatus

- Inclined plane with pivot (C)
- Cart (400 g + extra weights 100 g and 150 g)
- Pulley + hanging mass (80 g)

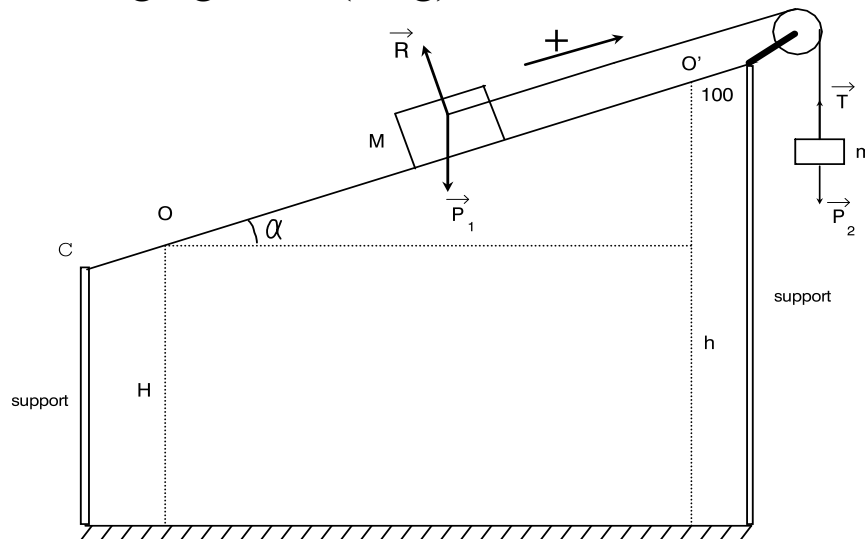


Figure 5 : Equilibrium setup

Procedure

1. Balance the system.
2. Adjust inclination to find α_1 and α_2 .
3. Compute f_s from the equations.
4. Estimate error on f_s (neglect minimal terms).

Put the results in the following table.

	h	H	$\sin \alpha = \frac{h-H}{OO'}$
$\sin \alpha_1$			
$\sin \alpha_2$			

Determining $\sin \alpha_1$ and $\sin \alpha_2$ allows us to calculate the frictional force f_s using equations (1) and (2) from paragraph 2.2.

We take $g = (9.80 \pm 0.01) \text{ m/s}^2$

$$f_s = \frac{Mg(\sin \alpha_2 - \sin \alpha_1)}{2} \dots\dots\dots(3)$$

We calculate the error on f_s and consider relative errors less than 1/200 to be negligible.

$$\dots\dots\dots(4)$$

$$\Delta f_s = \frac{Mg}{2} [\Delta(\sin \alpha_1) + \Delta(\sin \alpha_2)]$$

$$\frac{\Delta(\sin \alpha_1)}{\sin \alpha_1} = \frac{\Delta h + \Delta H}{h_1 - H_1} + \frac{\Delta OO'}{OO'} \quad \text{et} \quad \frac{\Delta(\sin \alpha_2)}{\sin \alpha_2} = \frac{\Delta h + \Delta H}{h_2 - H_2} + \frac{\Delta OO'}{OO'}$$

$$\Delta f_s = \frac{Mg}{2} \left[\left(\frac{\Delta h + \Delta H}{h_1 - H_1} + \frac{\Delta OO'}{OO'} \right) \sin \alpha_1 + \left(\frac{\Delta h + \Delta H}{h_2 - H_2} + \frac{\Delta OO'}{OO'} \right) \sin \alpha_2 \right]$$

4. Experiment 2 – Dynamic Friction Force

Apparatus

- Inclined plane with photocells at A and B (AB = 90 cm).
- Stopwatch linked to photocells.

- Same cart as in experiment 1.

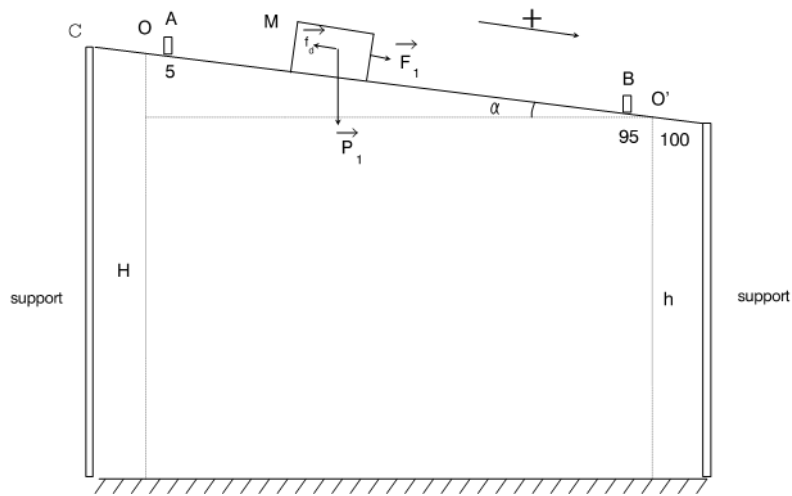


Figure 6. Uniformly accelerated motion.

Procedure

1. Place the cart at a (5 cm), release it, and measure travel time.
2. Repeat 5 times for each inclination.
3. Compute average time $\bar{t}$ and standard deviation σ .
4. Use :

$$f_d = Mg \sin \alpha - M\gamma$$

With $\gamma = \frac{2AB}{t^2}$

And from It :

$$f_d = M(g \sin \alpha - \frac{2AB}{t^2}) \dots\dots(5)$$

Size chart

H	h	t	t _{moy}	σ	Δt	sin α	Δ(sin α)	f _d	Δf _d			
mm	mm	s	s	s	s			N	N			
	270	t ₁	cal	cal				cal	cal			
		t ₂	≈	≈				≈	≈			
		t ₃										
		t ₄										
		t ₅										
	250	t ₁	cal	cal				cal	cal			
		t ₂	≈	≈				≈	≈			
		t ₃										
		t ₄										
		t ₅										
	230	t ₁	cal	cal				cal	cal			
		t ₂	≈	≈				≈	≈			
		t ₃										
		t ₄										
		t ₅										
	210	t ₁	cal	cal				cal	cal			
		t ₂	≈	≈				≈	≈			
		t ₃										
		t ₄										
		t ₅										

ΔAB		$\frac{\Delta AB}{AB}$	
Δg		$\frac{\Delta g}{g}$	
ΔM		$\frac{\Delta M}{M}$	

5. Error Analysis

We calculate the error on f_d , assuming that relative errors less than 1/200 are negligible. We re-calculate the error, but we cancel the negligible terms.

The expression for f_d in Equation 5 gives:

$$\frac{\Delta f_d}{f_d} = \frac{\Delta M}{M} + \frac{\Delta \left(g \sin \alpha - \frac{2AB}{t^2} \right)}{g \sin \alpha - \frac{2AB}{t^2}}$$

$$\frac{\Delta f_d}{f_d} = \frac{\Delta M}{M} + \frac{\Delta(g \sin \alpha)}{g \sin \alpha - \frac{2AB}{t^2}} + \frac{\Delta \left(\frac{2AB}{t^2} \right)}{g \sin \alpha - \frac{2AB}{t^2}}$$

$$\frac{\Delta f_d}{f_d} = \frac{\Delta M}{M} + \frac{\left(\frac{\Delta g}{g} + \frac{\Delta(\sin \alpha)}{\sin \alpha} \right) g \sin \alpha}{g \sin \alpha - \frac{2AB}{t^2}} + \frac{\left(\frac{\Delta AB}{AB} + \frac{2\Delta t}{t} \right) \frac{2AB}{t^2}}{g \sin \alpha - \frac{2AB}{t^2}}$$

$$\frac{\Delta f_d}{f_d} = \frac{\Delta M}{M} + \frac{\sin \alpha \Delta g + g \Delta(\sin \alpha) + \frac{2}{t^2} \Delta AB + \frac{4AB}{t^3} \Delta t}{g \sin \alpha - \frac{2AB}{t^2}}$$

6. Questions

- Keep only significant figures.
- Identify the largest error sources (time, angles, and lengths).

- Compare $\mathbf{f}_s$ and $\mathbf{f}_d$, including uncertainties.
- Discuss whether values are consistent.

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